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"The Lacunarity of Colour Fractal Images"

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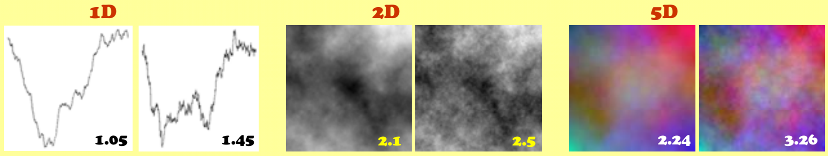
Fractal dimension and lacunarity are widely-used for the analysis of textures. However, all the existing approaches for computing these two measures are defined for one binary and gray-scale images. We propose a colour approach, derived from the existing probabilistic algorithm for the computation of the fractal dimension and lacunarity.

Random fractals (fBm)



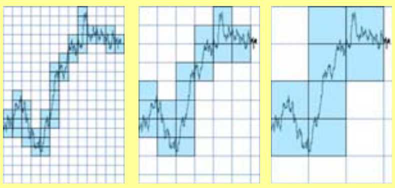
Fractional Brownian Motion (fBm)– the most used model for random / probabilistic fractal generation. We extended the 2D algorithm to 5D – to obtain colour fractals

Lacunarity $\Lambda(\delta)$ is a mass distribution function by definition, a measure that characterizes the way in which the fractal set occupies the available topological space.

According to the definition of Voss, lacunarity is the "entropy" of the points of the discrete surface representing the image, in other words the "dance" of the luminosity on the z axis. In our colour approach, lacunarity characterizes the spread of vectors in the RGB space and represents a measure of the correlation between colours represented in the RGB colour model.

Box-Counting FD estimation for 1D fractals:

- for different box-sizes (δ), count how many boxes ($N(\delta)$) are needed to cover the object
- then the FD is estimated as the slope of the regression line through the points <log(δ),-log($N(\delta)$)>.



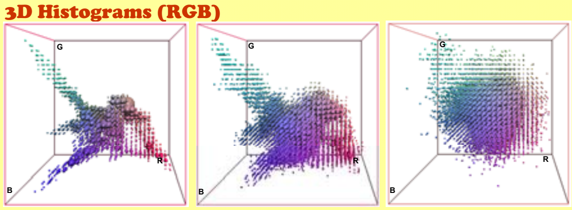
δ=3, $N(\delta)$=48 δ=5, $N(\delta)$=19 δ=7, $N(\delta)$=8

Our approach is a colour extension of the Voss probabilistic (box-counting like) method. We consider the colour images as 5D objects (pixel's spatial coordinates + colour: Red (R), Green (G) & Blue (B)) and, instead of boxes, we use hyper-cubes in the (x,y,R,G,B) space.

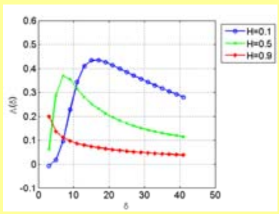
VALIDATION

Synthetic Colour Fractal Images

3D Histograms (RGB)



3D histograms reveal the complexity in RGB space

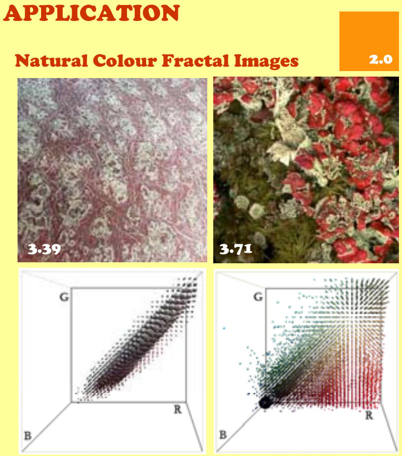


Corresponding lacunarity curves:

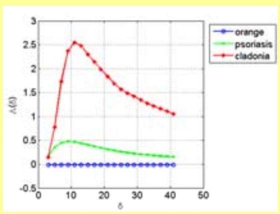
- Higher complexity ⟶ Larger lacunarity
- Correct ranking

APPLICATION

Natural Colour Fractal Images

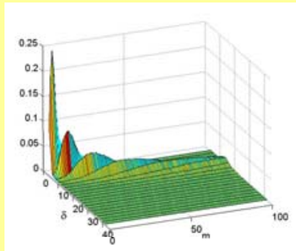


3D histograms (RGB)



The corresponding colour lacunarity curves reveal the real complexity and the correct ranking.

P(m,δ) is the probability of having m points inside a cube of size δ (box)



P(m,δ) for the H=0.9 colour fractal

The total number of boxes needed to cover the image is:

$$\langle N(\delta)\rangle = \sum_{m=1}^{N} \frac{M}{m} P(m,\delta) = M \sum_{m=1}^{N} \frac{1}{m} P(m,\delta)$$

$N(\delta)$ is proportional to L^{-FD}.

Mandelbrot introduces the term lacunarity and suggested several definitions, e.g.

$$\Lambda = E\left(\left(\frac{M}{E(M)} - 1\right)^2\right)$$

The Voss definition of lacunarity is based on P(m, δ) and the two statistical moments:

$$\Lambda(\delta) = \frac{M^2(\delta) - (M(\delta))^2}{(M(\delta))^2}$$

$$M(\delta) = \sum_{m=1}^{N} m P(m,\delta)$$

$$M^2(\delta) = \sum_{m=1}^{N} m^2 P(m,\delta)$$

Our colour extension allows thus computing the lacunarity for colour fractal images as well, according to the Voss definition.

There exist other definitions:

$$C(\delta) = \frac{M(\delta) - N(\delta)}{M(\delta) + N(\delta)}$$

We proposed a color approach for the computation of lacunarity, as an extension of the classical probabilistic algorithm, widely used to compute the fractal dimension of images. Our purpose was to develop a texture analysis tool for colour images based on fractal measures. The lacunarity we computed for the colour fractal images reflects the complexity in the RGB space, despite the fact that apparently the images may seem visually more complex. The perceived complexity of a colour texture can be lower than the one revealed by a marginal analysis. Because the great majority of images are uncalibrated and stored without information about the illuminant or the acquisition conditions, the best color space for analysis is the RGB space or another derived by linear space transformation. We made the choice of image representation in the RGB space due to the coherence between the generation method and the analysis approach. Nevertheless, this choice is not a constraint and the fractal measures can be expressed by using other colour spaces, the only issue being then the distance expression between the colours and the spatial coordinates.